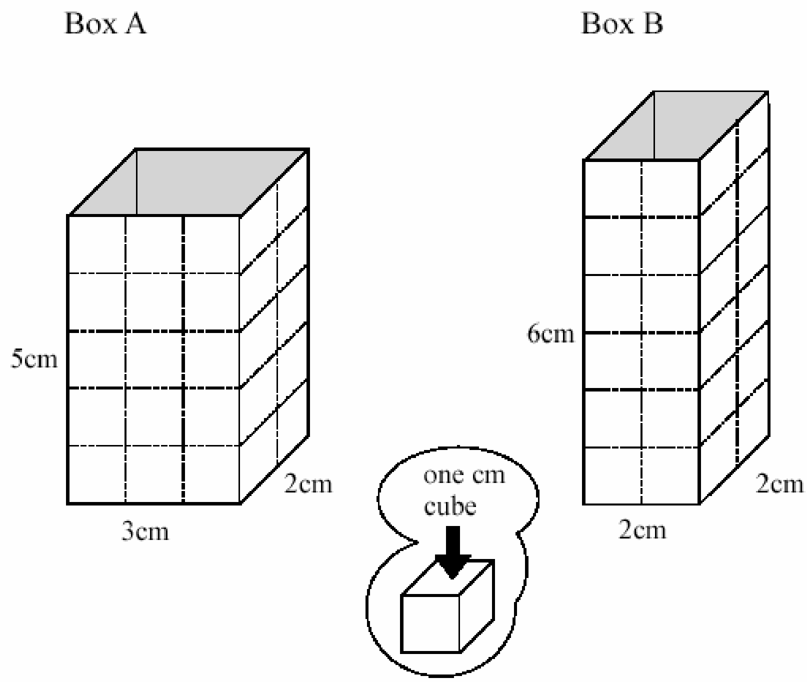
How Many Cubes?

This problem gives you the chance to:
- work with volume

Steve fills Box A and Box B with one centimeter cubes.



1. How many cubes can Steve fit into Box A? ________________

   Explain how you figured it out.

2. How many cubes can Steve fit into Box B? ________________

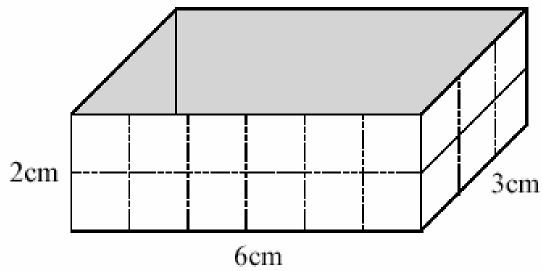
   Show your calculations.

3. Which of the two boxes can hold more cubes? ________________

4. Here is another box.

How many centimeter cubes can this box hold?

____



Find the measurements of a different box that holds the same number of cubes as this box.

__________ cm long __________ cm wide __________ cm high

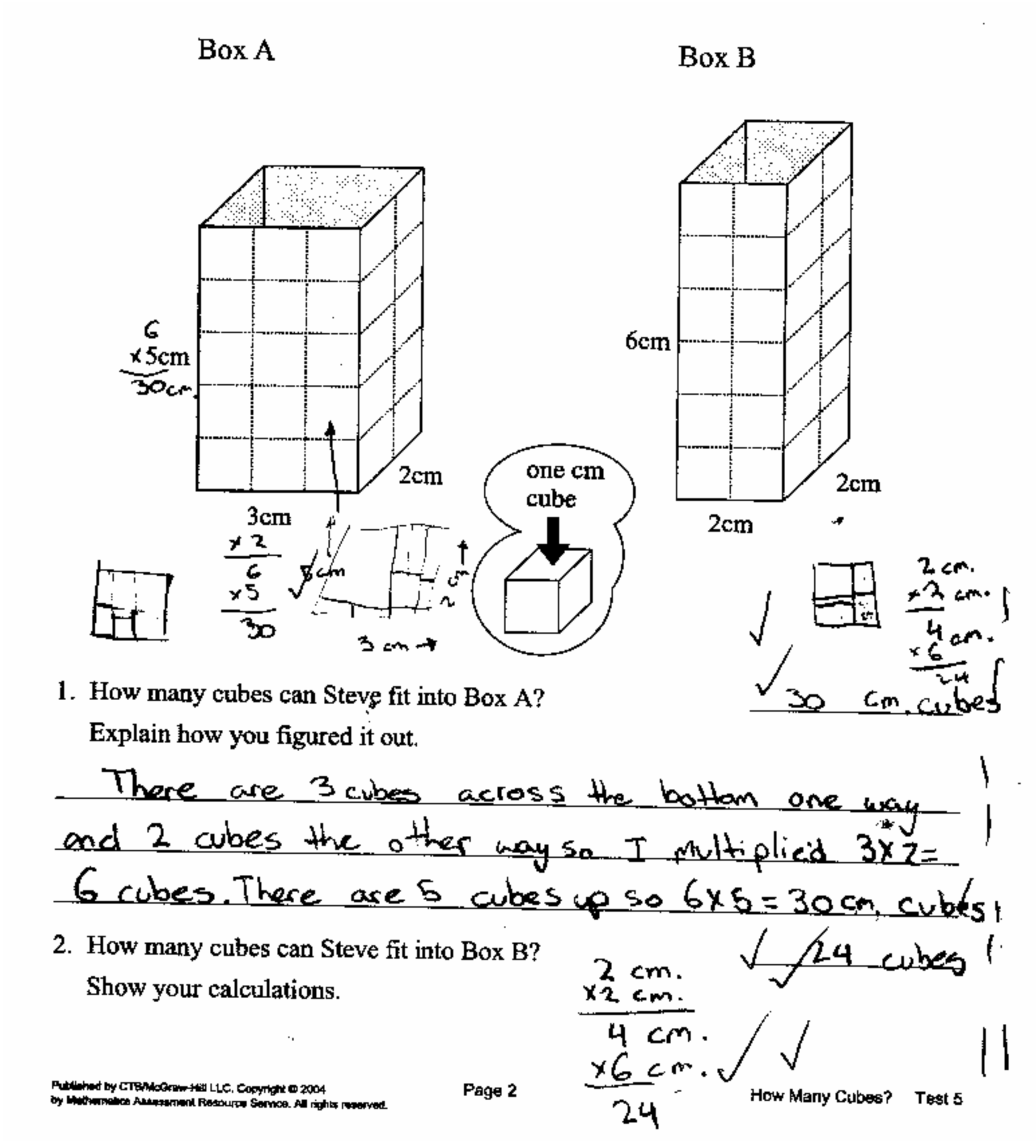
7

How Many Cubes? Test 5 Rubric		
The core elements of performance required by this task are: • work with volume Based on these, credit for specific aspects of performance could be assigned as follows	points	section points
1. Gives correct answer: **30** cubes Gives a correct explanation such as: There are 6 cubes on each layer and 5 layers. 3 x 2 x 5 =	1 1	2
2. Gives correct answer: **24** cubes Shows work such as: 2 x 2 x 6 =	1 1	2
3. Gives correct answer: Box A	1 ft	1
4. Gives correct answer: **36** cubes Gives a correct answer such as: 4 x 3 x 3 **or** 2 x 2 x 9 Do not accept boxes with a 6 x 3 or 3 x 6 base.	1 1	2
Total Points		7

Looking at Student Work – How Many Cubes?

Many students did well on how many cubes. Students with a perfect score showed several strategies for making sense of the problem. Student A makes diagrams of the bottom layer of each of the boxes and then thinks about the number of layers needed to build the box. This thinking of the bottom layer is even shown in the way the measurements for the new box are designed in part 4.

Student A



Box A

Box B

6 ×5cm 30cm

6cm

2cm

one cm cube

2cm

3cm ×2 6 ×5 30

5cm

3 cm

2cm

2 cm. ×2 cm. 4 cm. ×6 24

1. How many cubes can Steve fit into Box A? 30 cm. cubes

Explain how you figured it out.

There are 3 cubes across the bottom one way and 2 cubes the other way so I multiplied 3×2= 6 cubes. There are 5 cubes up so 6×5=30 cm. cubes

2. How many cubes can Steve fit into Box B? 24 cubes

Show your calculations.

2 cm. ×2 cm. 4 cm. ×6 cm. 24

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How Many Cubes? Test 5

Student A, part 2

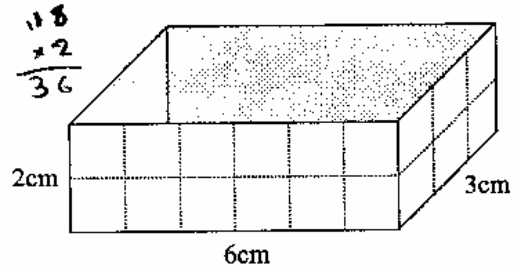
3. Which of the two boxes can hold more cubes? Box A ✓

4. Here is another box.

How many centimeter cubes can this box hold?

36 cubes ✓

3 × 6 = 18
18 × 2 = 36



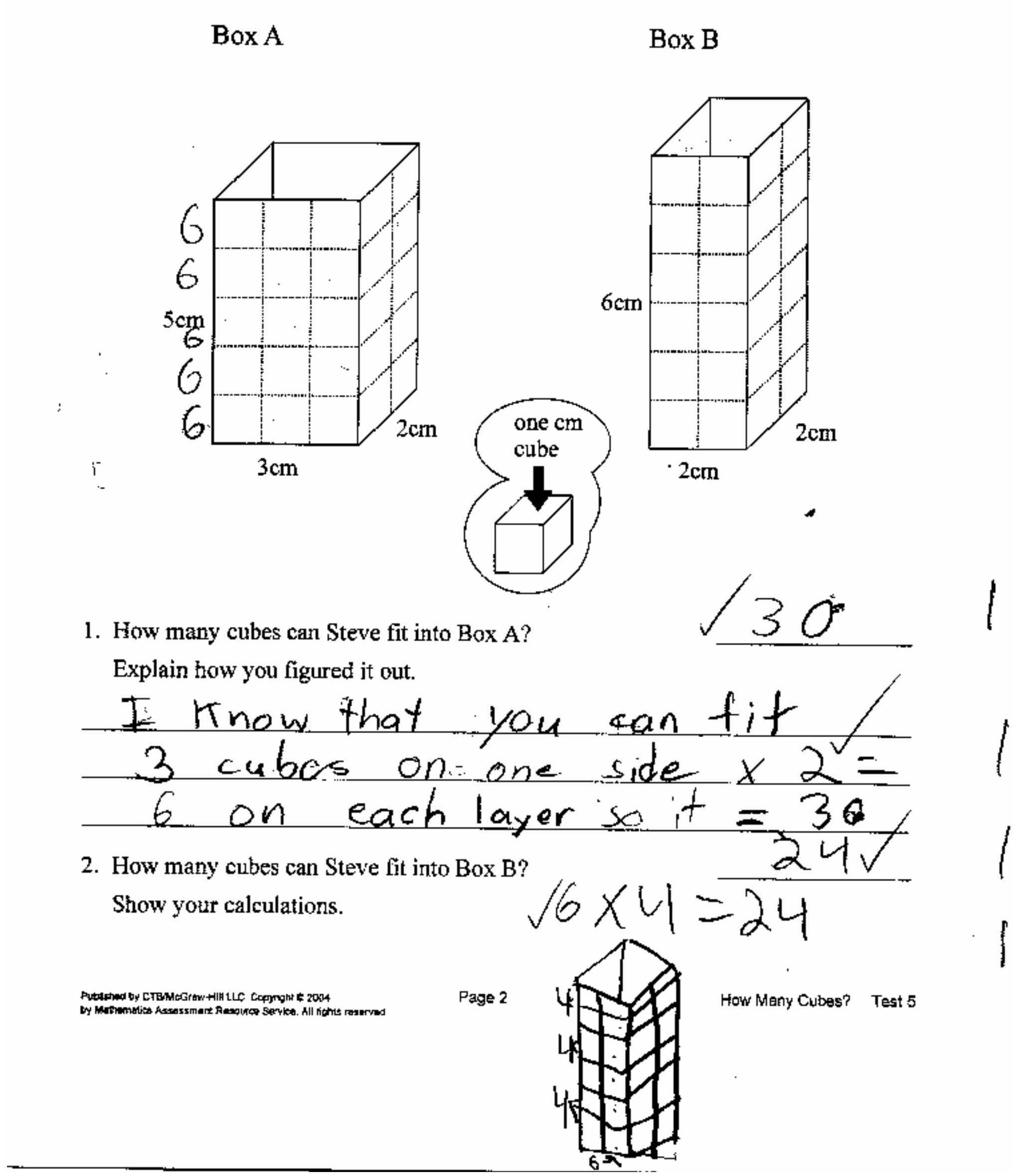
Find the measurements of a different box that holds the same number of cubes as this box.

2 cm long   3 cm wide   6 cm high ✓✓

3cm × 2 cm. = 6 cm

6 × 6 = 36 cm

Student B also shows that counting of layers to fill the box.


Box A

Box B

6
6
5cm 6
6
6

2cm

3cm

one cm cube

6cm

2cm

2cm

1. How many cubes can Steve fit into Box A? √30 1

Explain how you figured it out.

I Know that you can fit 3 cubes on one side x 2 = 6 on each layer so it = 30 1 1

2. How many cubes can Steve fit into Box B? 24√ 1

Show your calculations. √6 X 4 = 24 1

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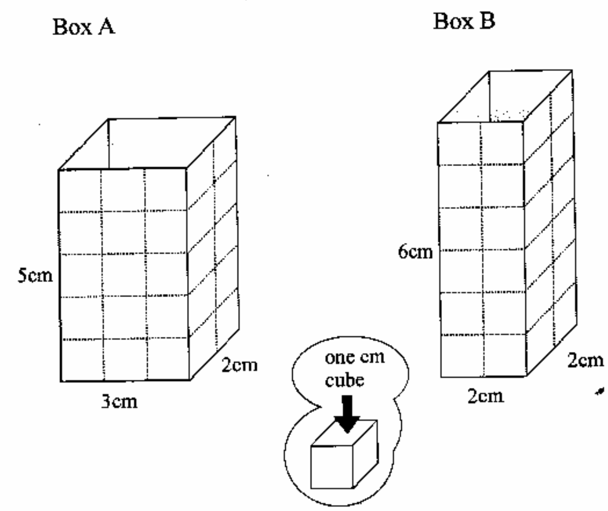
How Many Cubes? Test 5

4
4
4
6

Other students seemed to know a formula of multiplying length times width times height. Student C uses the formula strategy. Also note that student B is able to make a model of the new box in part 4.



1. How many cubes can Steve fit into Box A? ____30 cubes____

   Explain how you figured it out.

   Knowing the length was 3cm and the width was 2cm, I multiplied 3cm by 5cm, which was the height. It was 15 cm. The shape's width was 2cm., so 15x2 =30 cubes.

2. How many cubes can Steve fit into Box B? ____24 cubes____

   Show your calculations.

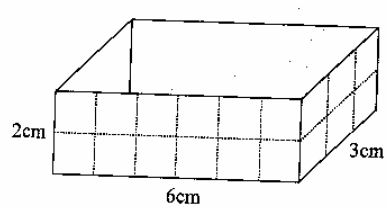
   6x2 = 12
   12x2=24 cubes

4. Here is another box.

2x6 =12cm 12 x 3 = 36 cubes

How many centimeter cubes can this box hold?

____36 cubes____

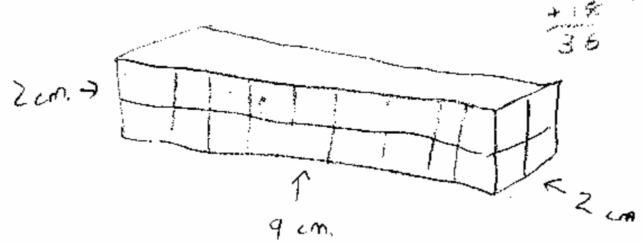


Find the measurements of a different box that holds the same number of cubes as this box.

____9____ cm long ____2____ cm wide ____2____ cm high

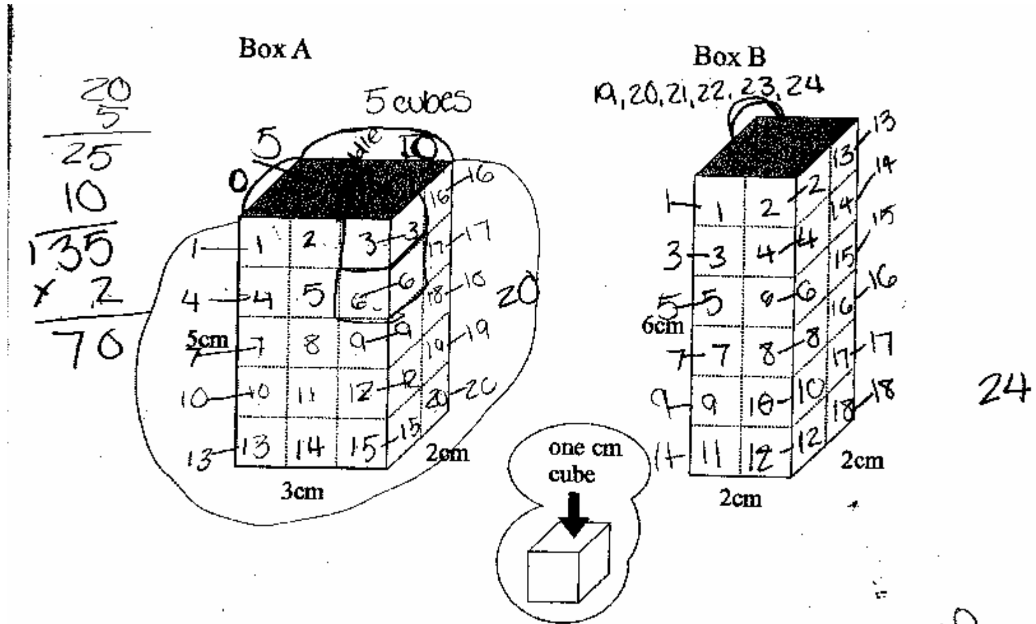
2x9 = 18. 18 x2 = 36 cube

18 + 18 = 36



Some students struggled with interpreting the diagrams in the task. Student D is starting to develop an understanding of how cubes fit into a three-dimensional shape. The student knows that the squares on the corner represent the same cube. The student also knows that there are some "hidden" cubes that are not shown in diagram, such as a middle row of five at the back of box A. However Student D does not have a complete strategy for counting each cube only one time.

Student D



1. How many cubes can Steve fit into Box A? 70 cubes

Explain how you figured it out.

First I labeled the squares and I did it so that I counted the squares on the corners as one square. then I multiplied it by two because of the inside.

2. How many cubes can Steve fit into Box B? 48 cubes

Show your calculations.

24 × 2 = 48

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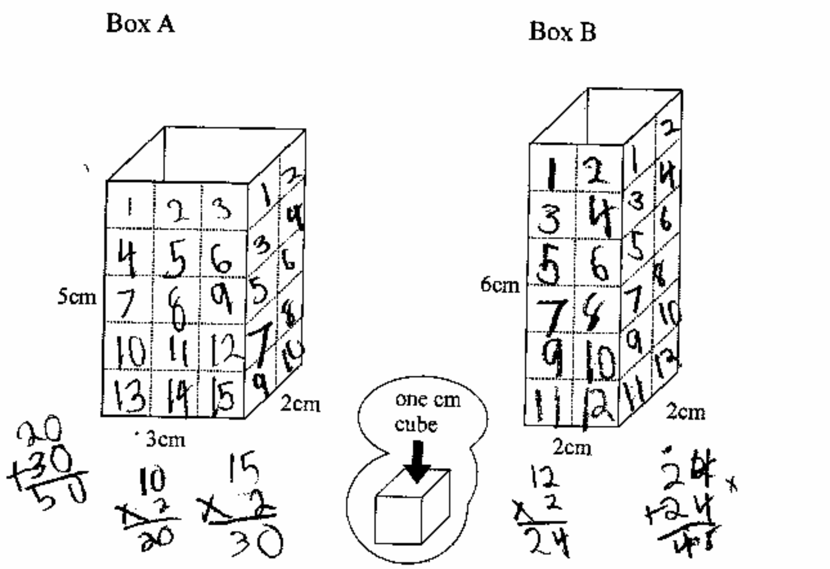
Many students have not grasped the idea that corner squares are represented by a front and side square in the diagram or the idea of hidden squares in the middle of the box. Student E thinks only about the cubes touching the exterior of the box. So there are fifteen cubes in the front and back for box A and ten cubes on each side. Thus the student counts some cubes twice and ignores cubes not touching the exterior. The student drawing, which is quite common, also shows the student as thinking about exterior squares rather than a three-dimensional object.

Student E



1. How many cubes can Steve fit into Box A? 50 x 0

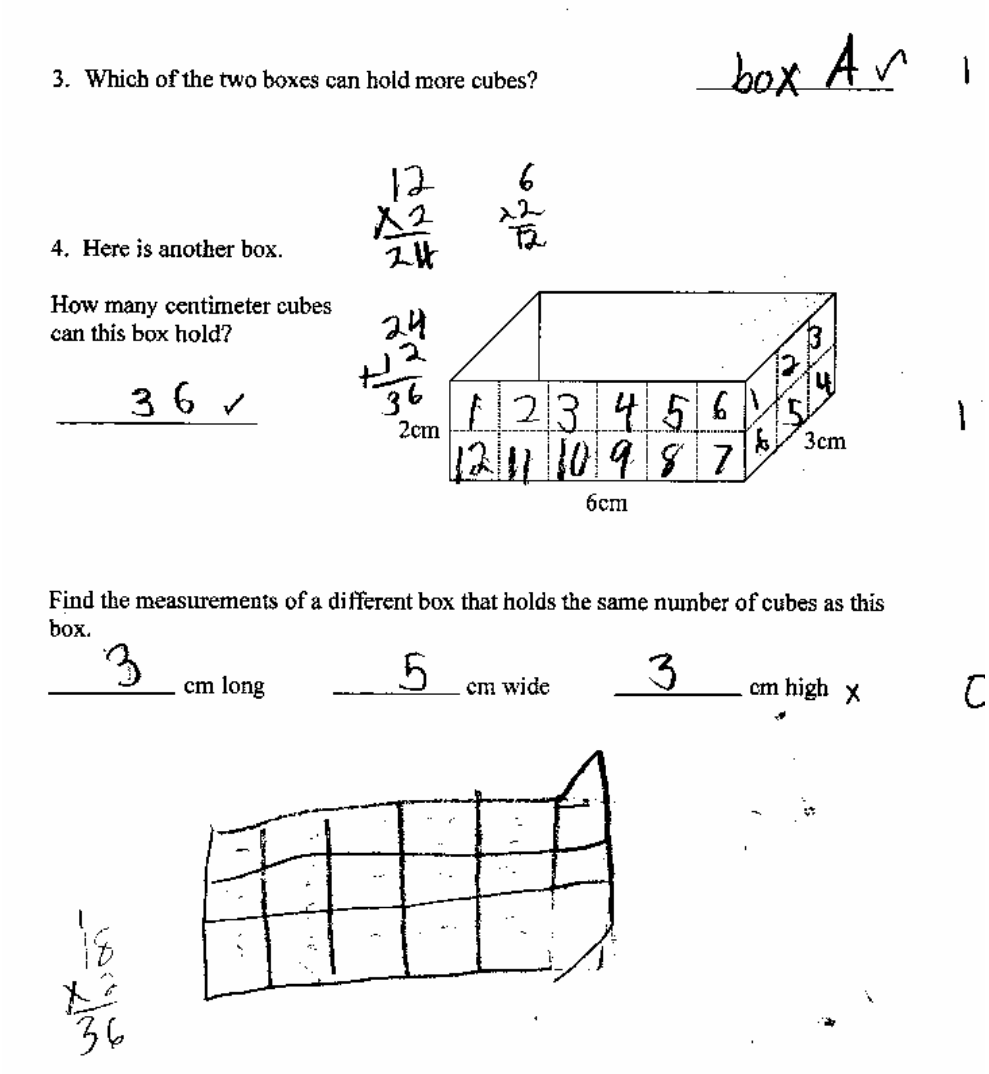
   Explain how you figured it out.

   I counted 15 squares in front and x it by 2 and did the same with the side x 0

2. How many cubes can Steve fit into Box B? 48 x 0

   Show your calculations.

Student E, part 2



3. Which of the two boxes can hold more cubes? box A ✓ 1

4. Here is another box.

How many centimeter cubes can this box hold?

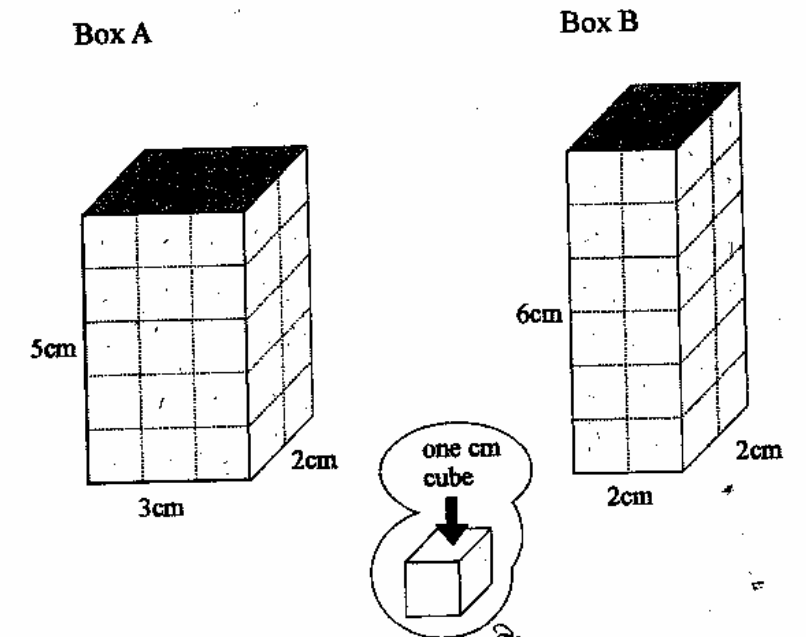
36 ✓ 1

Find the measurements of a different box that holds the same number of cubes as this box.

3 cm long 5 cm wide 3 cm high x C

Student F also struggles with spatial visualization and interpreting three-dimensional objects. The student does not recognize that the front and side of Box A are of different sizes. The student finds the squares in one side and multiplies by 4 sides, which also ignores the possibility of interior cubes and the idea that some cubes are shown on the front and side represent the same cube.

Student F



1. How many cubes can Steve fit into Box A? → 15 ×4 = 60   X 60 cubes 0

   Explain how you figured it out.

   I counted how much cube are on one side and multiplied it X by how many side that are on the box. 0

2. How many cubes can Steve fit into Box B?   X 48 cubes 0

   Show your calculations. 12 → how many cubes on one side
   How many side there are ← ×4 X
   48   (

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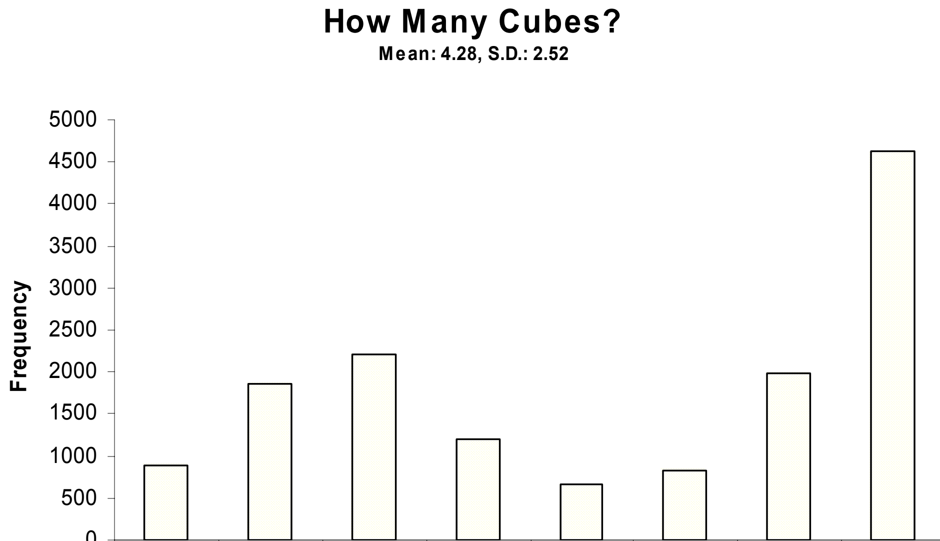
Teacher Notes:

Frequency Distribution for each Task – Grade 5
Grade 5 – How Many Cubes?



	0	1	2	3	4	5	6	7
Frequency	883	1852	2202	1193	670	824	1984	4628

Score:	0	1	2	3	4	5	6	7
% < =	6.2%	19.2%	34.7%	43.1%	47.8%	53.6%	67.5%	100.0%
% > =	100.0%	93.8%	80.8%	65.3%	56.9%	52.2%	46.4%	32.5%

The maximum score available for this task is 7 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students (about 90%) could identify the larger box. Many students (about 81%) could identify the larger box and find the cubes needed to fill either box B or the box in part 4. More than half the students (about 59%) could find the number of cubes in Box B with the correct calculations, identify the larger box, and find either the correct number of cubes in the box for part 4 or give dimensions for a box that would hold the same number of cubes. More than 30% of the students could meet all the demands of the task. About 8% of the students scored no points on this task. More than 83% of the students with this score attempted the task.

How Many Cubes?

Points	Understandings	Misunderstandings
0	About 83% of the students with this score attempted the problem.	Students tended to count exterior squares on the diagrams rather than thinking in 3-dimensions. This led to double counting and ignoring cubes with no exterior sides.
1	Students could identify the larger box.	Students with this score miscalculated the number of cubes in box A and B and are only identifying a larger number.
2	Students could identify the larger box and find either the number of cubes in Box B or the box in part 4.	Often students used an incorrect method for finding the cubes in the box, although they may have reached the correct answer. For example Student F counts the number of squares in the front and back of box 4 and for the two sides to get a correct answer. But the process is about surface area and has nothing to do with thinking in 3-dimensions. Students with this score also had trouble trying to make 3-dimensional diagrams to help them think about part 4.
4	Students could identify the larger box, find the correct number of cubes for box b with a correct process, and either find the number of cubes in box 4 or give dimensions for a box with the same volume.	In part one many students found the squares on the 4 exterior sides of the cubes, giving them a total of 50 for box A and 48 for box B. This accounts for almost 50% of the errors. Another error is to just count the visible, white squares in the diagrams. This would give a total of 25 and 24 for boxes A and B. This represents about 30% of the errors.
6	Students could correctly calculate the number of cubes in boxes A and B and identify the larger box. They could also either find the number of cubes in part 4 or give the dimensions of another box with the same volume.	The most common error for part 4 was 18 cubes. Some students used a rotation of the given box, when trying to design their own box in part 4.
7	Students had a correct strategy for finding the number of cubes in a box, by counting the cubes in a layer and thinking about the number of layers or using the formula.	

Based on teacher observations, this is what fifth graders knew and were able to do:

- Know a formula for finding volume or were able to derive their own formula
- Understand that there were more cubes in the box than the ones showing in the diagram
- Design a box with same volume but different dimensions

Areas of difficulty for fifth graders, fifth grade students struggled with:

- Differentiating between volume and surface area
- Understanding that more than one square in the diagram might represent the same cube (how to interpret a 3-dimensional diagram)

Teacher Notes:

Questions for Reflection on How Many Cubes?

- What types of experiences have your students had building three-dimensional shapes?
- What opportunities have students had drawing 3-dimensional shapes? Free-hand, in perspective, using isometric dot paper?
- What kinds of questions or experiences help students to develop a sense of building shapes in layers and relating the layers to relevant parts of diagrams?
- What kinds of questions or experiences help students to see that a cube may have 1, 2, 3, or no faces shown in a diagram?
- When you look at student-made diagrams in this task, are there the same number of layers on the front and side? Do students attempt to show a three-dimensional figure or do they just show two sides?

How many of your student for box A, gave answers of:

30	50	25	60	10 or 15	other

What types of reasoning led them to these answers? What does this show they are not considering in the diagrams or don't understand about 3-dimensional drawings? Do they have any ideas about filling or is their reasoning focused on some form of surface area counting?

Implications for Instruction:

Students at this level need to have many experiences building 3-dimensional shapes and making diagrams to represent these shapes. Students should also be able to use a diagram to build the shape it represents. Students need to be asked questions to help them see that cubes may have1, 2, 3, or 0 faces shown in a diagram. They also need experiences with filling shapes with cubes and developing ideas about how the shape is built by making layers. This ability to be fluent between diagrams and physical models is a critical foundation for the mathematics they will be learning at the middle grades and for understanding and applying geometrical formulas.

Teacher Notes:

Performance Assessment Task

How Many Cubes?
Grade 5

This task challenges a student to use knowledge of concept of volume and its attributes to solve problems with rectangular prisms by counting, addition, or using multiplication. A student must be able to calculate and compare volumes for different solids. A student must be able to use knowledge about the concept of volume to construct or design a rectangular prism with a given volume.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume and measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volume, e.g., to represent the associative property of multiplication.
b. Apply the formulas V= l x w x h and V = b x h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about

the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
5	2004	7	4	65%